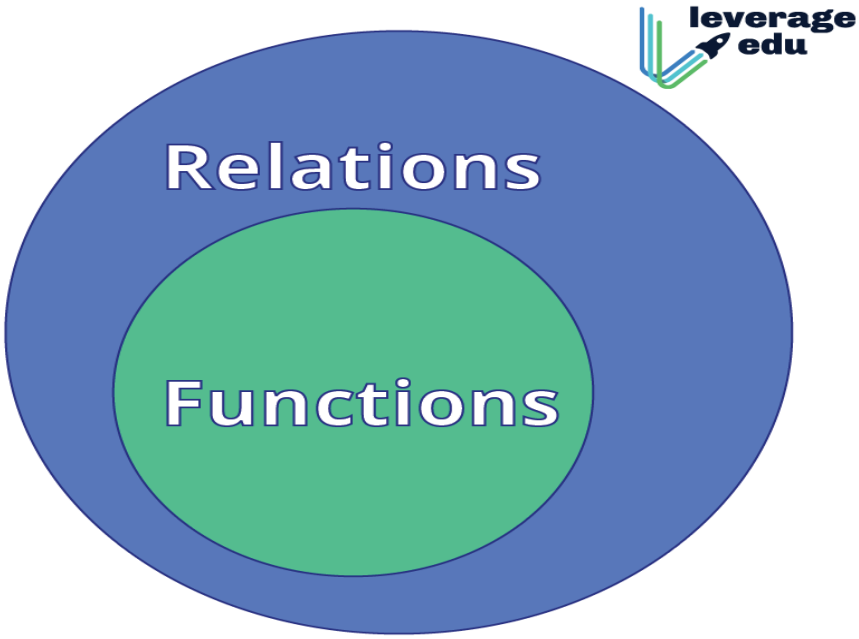
Relation and Function Theory

In the relation and function theory, we always represent an ordered pair as (INPUT, OUTPUT). In this, the Relation shows the relationship between INPUT and OUTPUT. On the other hand, a function represents a relation which is derived through one OUTPUT for every single INPUT.



All the functions are relations, but not all the relations are functions.

Now that you are familiar with the theory of relations and functions, let's focus on the definition, types and examples to understand them better.

What is a Function?

A function can be described as a relation which represents that there should be only one output for every input. In other words, functions can be understood as a unique relation which follows the rule i.e., each and every value of X should be related to only one value of Y, then it will be called as a function.

A function comprises of Domain and Range. A Domain is a collection of first values in the ordered pair i.e., it is the set of all the inputs of the X variable. Whereas Range refers to the collection of the second values in the ordered pair i.e., it is the output of all the Y variables.

Domain	Range
-1	-3
1	3
3	9

Example of Functions:

For instance, the Relation is {(-2,3), (4,5), (6,-5), (-2,3)}.

Then, the Domain would be {-2, 4,6 } and {-5,3,5}.

Note: If any number appears more than once, it will only be written once in the domain as well as range.

Types of Functions

Now that you are familiar with its main concept, mentioned below are some types of functions you must know about to understand relations and functions in a better way.

Injective Function or One to One Function: In a function f: P→Q is considered to be One to One function only if every element of P there is a distinct element of Q.

Many to One Function: A function in which two or more elements of set P are mapped to the same element of set Q.

Surjective Function or Onto Function: It refers to a function where for every element of set Q there is a pre-image in set P.

Bijective Function or One-One and Onto Function: In the function f, each element of P is matched with a discrete element of Q and there is a preimage of every element of Q in P.

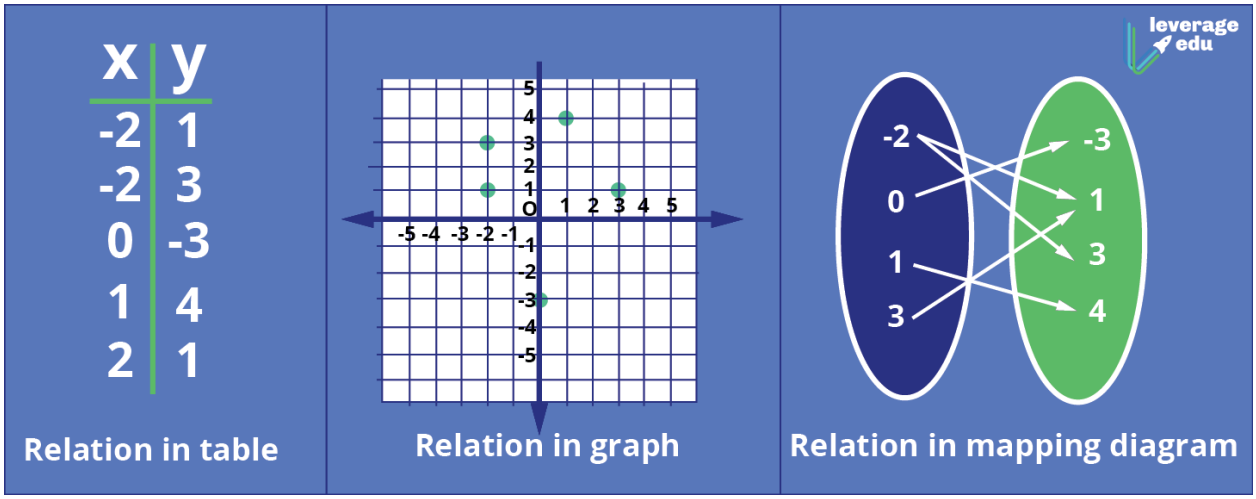
What are Relations?

Once you are familiar with functions, then it will be easier to grasp the concept of relations. In simple terms, relations can be simply understood as a subset of the Cartesian product or a bunch of points in an ordered pair.

For example: {(-2,1), (4,3), (7,-3)}

Representation of Relations

Apart from the common set notation, there are many other ways of representing a relation. Popular ways to do so are using tables, mapping diagram or plotting it on the XY axis.



Types of Relations

Similar to the types of functions, there are various relations through which we can learn about their different properties and these are:

- Universal Relations
- Empty Relations
- Inverse Relations
- Reflexive Relations
- Identity Relations
- Transitive Relations
- Symmetric Relations

Solved Examples

Example 1: Identify the range and domain the relation below: {(-2, 3), {4, 5), (6, -5), (-2,3)}

Solution: Since the x values are the domain, the answer is, therefore,

$\Rightarrow$ {-2, 4, 6}

The range is {-5, 3, 5}.

Example 2: Check whether the following relation is a function: B = {(1, 5), (1, 5), (3, -8), (3, -8), (3, -8)}

Solution: B = {(1, 5), (1, 5), (3, -8), (3, -8), (3, -8)}

Though a relation is not classified as a function if there is a repetition of x – values, this problem is a bit tricky because x values are repeated with their corresponding y-values.

Example 3: Determine the domain and range of the following function: Z = {(1, 120), (2, 100), (3, 150), (4, 130)}.

Solution: Domain of z = {1, 2, 3, 4 and the range is {120, 100, 150, 130}

Example 4: Check if the following ordered pairs are functions:

1. W= {(1, 2), (2, 3), (3, 4), (4, 5)
2. Y = {(1, 6), (2, 5), (1, 9), (4, 3)}

Solution:

1. All the first values in W = {(1, 2), (2, 3), (3, 4), (4, 5)} are not repeated, therefore, this is a function.
2. Y = {(1, 6), (2, 5), (1, 9), (4, 3)} is not a function because the first value 1 has been repeated twice.

Example 5: Determine whether the following ordered pairs of numbers are a function. R = (1,1); (2,2); (3,1); (4,2); (5,1); (6,7)

Solution: There is no repetition of x values in the given set of ordered pairs of numbers.

Therefore, R = (1,1); (2,2); (3,1); (4,2); (5,1); (6,7) is a function.