

MATHEMATICS (860)

Aims:

1. To enable candidates to acquire knowledge and to develop an understanding of the terms, concepts, symbols, definitions, principles, processes and formulae of Mathematics at the Senior Secondary stage.
2. To develop the ability to apply the knowledge and understanding of Mathematics to unfamiliar situations or to new problems.
3. To develop skills of -
 - (a) computation.
 - (b) reading tables, charts, graphs, etc.
4. To develop an appreciation of the role of Mathematics in day-to-day life.
5. To develop an interest in Mathematics.
6. To develop a scientific attitude through the study of Mathematics.

A knowledge of Arithmetic and Pure Geometry is assumed. The parts of Geometry which are of chief importance in other branches of Mathematics are the fundamentals concerning angles, parallels (including

lines and planes in space), similar triangles (including the theorem of Pythagoras) the 'symmetry' properties of chords and tangents of a circle, and the theorem that a line perpendicular to two non-parallel lines in a plane is perpendicular to every line therein. The examination may include questions with a geometrical content.

As regards the standard of algebraic manipulation, students should be taught:

- (i) To check every step before proceeding to the next particularly where minus signs are involved.
- (ii) To attack simplification piecemeal rather than en block, e.g. never to keep a common factor which can be cancelled.
- (iii) To observe and act on any special features of algebraic form that may be obviously present.

The standard as regards (iii) is difficult to define; initial practice should be on the easiest cases, 'trick' examples should be avoided and it should be kept in mind that (iii) is subsidiary in importance to (i) and (ii) Teachers should be scrupulous in setting a standard of neatness and in avoiding the slovenly habit of omitting brackets or replacing them by dots.

CLASS XI

The syllabus is divided into **three** sections A, B and C. Section A is compulsory for all candidates. Candidates will have a choice of attempting questions from **either** Section B or Section C.

There will be one paper of **three** hours duration of 100 marks.

Section A (80 marks) will consist of nine questions. Candidates will be required to answer **Question 1 (compulsory)** and **five** out of the rest of the **eight** questions.

Section B / C (20 marks) Candidates will be required to answer **two** questions out of **three** from either Section B or Section C.

SECTION A

1. Mathematical Reasoning

Mathematically acceptable statements. Connecting words / phrases – consolidating the understanding of “if and only if (necessary and sufficient) condition”, “implies”, “and/or”, “implied by”, “and”, “or”, “there exists” and their use through variety of examples related to real life and Mathematics. Validating the statements involving the connecting words, differences between contradiction, converse and contrapositive.

2. Algebra

(i) Complex Numbers

- Complex numbers as an ordered pair of real numbers in the form $a + ib$, (a, b)
- Geometrical representation in complex plane - Argand diagram for z (a complex number), $1/z$, z and \bar{z} ; equality of two complex numbers; absolute value (modulus).

NOTE: Real and imaginary parts of a complex number and equality of two complex numbers are required to be covered.

(ii) Quadratic Equations

- Use of the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

in solving quadratic equations.

- **Equations reducible to quadratic form.**

• Nature of roots

- Product and sum of roots.
- Roots are rational, irrational, equal, reciprocal, one square of the other.
- Imaginary numbers.
- Complex roots.
- Framing quadratic equations with given root.

NOTE: Questions on equations having common roots are to be covered.

• Quadratic Functions.

- (i) Given α, β as roots then find the equation whose roots are of the form r^3, s^3 , etc.

- (ii) Case I: $a > 0$
- Real roots
 - Complex roots
 - Equal roots

- (iii) Case II: $a < 0$
- Real roots
 - Complex roots
 - Equal roots

where 'a' is the coefficient of x^2 in the equations of the form $ax^2 + bx + c = 0$.

Understanding the fact that a quadratic expression (when plotted on a graph) is a parabola.

- Sign of quadratic

Sign when the roots are real and when they are complex.

- Quadratic inequalities.

Using method of intervals for solving problems of the type:

(a) $x^2 + x - 6 \geq 0$

(b) A perfect square e.g. $x^2 - 6x + 9 \geq 0$

(c) $\frac{3x^2 - 2x - 5}{x^2 - 2x + 5} < 2$

(d) $\frac{x^2 - 2x + 5}{3x^2 - 2x - 5} > \frac{1}{2}$

(iii) Finite and Infinite Sequences

(a) Arithmetic Progression (A.P.)

- $T_n = a + (n - 1)d$
- $S_n = \frac{n}{2} \{2a + (n - 1)d\}$
- Arithmetic mean: $2b = a + c$
 - Inserting 2 or 3 arithmetic mean between any two numbers.
 - Three numbers in A.P. $\therefore a - d, a, a + d$
 - Four in A.P.: $a - 3d, a - d, a + d, a + 3d$

(b) Geometric Progression (G.P.)

- $T_n = ar^{n-1}, S_n = \frac{a(r^n - 1)}{r - 1},$

$$S_\infty = \frac{a}{1 - r}; |r| < 1$$

- Geometric Mean, $b = \sqrt{ac}$
- Inserting 2 or 3 Geometric Mean between any two numbers.

- Three numbers are in G.P. ar, a, ar^{-1}
- Four $ar^3, ar, ar^{-1}, ar^{-3}$

(c) Harmonic Progression

- a, b, c are in H.P then $1/a, 1/b, 1/c$ are in A.P.
- Harmonic mean $b = \frac{2ac}{a+c}$

(d) Arithmetico Geometric Series

- Identifying series as A.P. x G.P. (when we substitute $d = 0$ in the series, we get a G.P. and when we substitute $r = 1$ the A.P.)

(e) Special sums $\sum n, \sum n^2, \sum n^3$

- Using these summations to sum up other related expression.

(iv) Permutations Combinations

- Factorial notation $n!, n! = n(n-1)!$
- Fundamental principle of counting.

(a) Permutations

- ${}^n P_r$.
- Restricted permutation.
- Certain things always occur together.
- Certain things never occur.
- Formation of numbers with digits.
- Word building - repeated letters – No letters repeated.
- Permutation of alike things.
- Permutation of Repeated things.
- Circular permutation – clockwise counterclockwise – Distinguishable / not distinguishable.

(b) Combinations

- ${}^n C_r, {}^n C_n = 1, {}^n C_0 = 1, {}^n C_r = {}^n C_{n-r}, {}^n C_x = {}^n C_y$, then $x + y = n$ or $x = y$, ${}^{n+1} C_r = {}^n C_{r-1} + {}^n C_r$.
- When all things are different.

- When all things are not different.
- Division into groups - e.g. distinct groups, identical groups.
- Mixed problems on permutation and combinations.

(v) Mathematical induction

Using induction to prove various summations and divisibility.

NOTE: Problems on inequalities are not required.

(vi) Binomial Theorem

- (a) Significance of Pascal's triangle.
- (b) Binomial theorem (proof using induction) for positive integral powers,

$$\text{i.e. } (x + y)^n = {}^n C_0 x^n + {}^n C_1 x^{n-1} y + \dots + {}^n C_n y^n.$$

Simple direct questions based on the above.

- (c) Binomial theorem for negative or fractional indices

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots$$

When $|x| < 1$

- Simple questions on the application of the above.
- Finding the r th term for the above (T_r).
- Applying the theorem on approximations e.g. $(0.99)^8 = (1-0.01)^8$.

NOTE: Algebraic approximations are also to be covered.

(vii) Properties of Binomial Coefficients.

$$C_0 + C_1 + \dots + C_n = 2^n$$

$$C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$$

Simple problems involving application of the above.

NOTE: Questions on the product of coefficients of $(1+x)^n(x+1)^m$ are excluded.

3. Trigonometry

(i) Angles and Arc lengths

- Angles: Convention of sign of angles.
- Magnitude of an angle: Measures of Angles; Circular measure.
- The relation $S = r\theta$ where θ is in radians. Relation between radians and degree.
- Definition of trigonometric functions with the help of unit circle.
- Truth of the identity $\sin^2 x + \cos^2 x = 1$.

NOTE: Questions on the area of a sector of a circle are required to be covered.

(ii) Trigonometric Functions

- Relationship between trigonometric functions.
- Proving simple identities.
- Signs of trigonometric functions.
- Domain and range of the trigonometric functions.
- Trigonometric functions of all angles.
- Periods of trigonometric functions.
- Graphs of simple trigonometric functions (only sketches).

NOTE: Graphs of $\sin x$, $\cos x$, $\tan x$, $\sec x$, $\operatorname{cosec} x$ and $\cot x$ are to be included.

(iii) Compound and multiple angles

- Addition and subtraction formula: $\sin(A \pm B)$; $\cos(A \pm B)$; $\tan(A \pm B)$; $\tan(A + B + C)$ etc., Double angle, triple angle, half angle and one third angle formula as special cases.
- Sum and differences as products $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$, etc.
- Product to sum or difference i.e. $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ etc.

(iv) Trigonometric Equations

- Solution of trigonometric equations (General solution and solution in the specified range).
 - (a) Equations in which only one function of a single angle is involved e.g. $\sin 5\theta = 0$
 - (b) Equations expressible in terms of one trigonometric ratio of the unknown angle.
 - (c) Equations involving multiple and sub-multiple angles.
 - (d) Linear equations of the form $a \cos \theta + b \sin \theta = c$, where $|c| \leq \sqrt{a^2 + b^2}$ and $a, b \neq 0$

4. Calculus

(i) Basic Concepts of Relations and Functions

- (a) Ordered pairs, sets of ordered pairs.
- (b) Cartesian Product (Cross) of two sets, cardinal number of a cross product.

Relations as:

- an association between two sets.
 - a subset of a Cross Product.
- (c) Types of Relations: reflexive, symmetric, transitive and equivalence relation.
 - (d) Binary Operation.
 - (e) Domain, Range and Co-domain of a Relation.
 - (f) Functions:

- As special relations, concept of writing "y is a function of x" as $y = f(x)$.
- Types: one to one/ many to one, into/onto.
- Domain and range of a function.
- Composite function.
- Inverse of a function.
- Classification of functions.
- Sketches of graphs of exponential function, logarithmic function, mod function, step function.

(ii) Differential calculus

(a) Limits

- Notion and meaning of limits.
- Fundamental theorems on limits (statement only).
- Limits of algebraic and trigonometric functions.

NOTE: Indeterminate forms are to be introduced while calculating limits.

(b) Continuity

- Continuity of a function at a point $x = a$.
- Continuity of a function in an interval.
- Removable discontinuity.

(c) Differentiation

- Meaning and geometrical interpretation of derivative.
- Concept of continuity and differentiability of $|x|$, $[x]$, etc.
- Derivatives of simple algebraic and trigonometric functions and their formulae.
- Differentiation using first principles.
- Derivatives of sum/difference.
- Derivatives of product of functions.
- Derivatives of quotients of functions.
- Derivatives of composite functions.

NOTE:

1. Derivatives of composite functions using chain rule.
2. All the functions above should be either algebraic or trigonometric in nature.

(d) Application of derivatives

- Equation of Tangent and Normal approximation.
- Rate measure.
- Sign of derivative.
- Monotonicity of a function.

(iii) Integral Calculus

Indefinite integral

- Integration as the inverse of differentiation.
- Anti-derivatives of polynomials and functions $(ax + b)^n$, $\sin x$, $\cos x$, $\sec^2 x$, $\operatorname{cosec}^2 x$.
- Integrals of the type $\sin^2 x$, $\sin^3 x$, $\sin^4 x$, $\cos^2 x$, $\cos^3 x$, $\cos^4 x$.

5. Coordinate Geometry

(i) Basic concepts of Points and their coordinates.

(ii) The straight line

- Slope and gradient of a line.
- Angle between two lines.
- Condition of perpendicularity and parallelism.
- Various forms of equation of lines.
- Slope intercept form.
- Two point slope form.
- Intercept form.
- Perpendicular /normal form.
- General equation of a line.
- Distance of a point from a line.
- Distance between parallel lines.
- Equation of lines bisecting the angle between two lines.
- Definition of a locus.
- Methods to find the equation of a locus.

(iii) Circles

Equations of a circle in:

- Standard form.
- Diameter form.
- General form.
- Parametric form.
- Given the equation of a circle, to find the centre and the radius.

- Finding the equation of a circle.
 - Given three non collinear points.
 - Given other sufficient data that the centre is (h, k) and it lies on a line and two points on the circle are given.
- Tangents:
 - Tangent to a circle when the slope of the tangent is given:

$$y = mx \pm a\sqrt{1 + m^2}$$
- Intersection:
 - Circle with a line hence to find the length of the chord.
- Finding the equation of a circle through the intersection of two circles i.e. $S_1 + kS_2 = 0$.

NOTE: Orthogonal circles are not required to be covered.

6. Statistics

- Measures of central tendency.
- Standard deviation - by direct method, short cut method and step deviation method.
- Combined mean and standard deviation.

NOTE:

1. Combined mean and standard deviation of two groups only are required to be covered.
2. Mean, Median and Mode of grouped and ungrouped data are required to be covered.

SECTION B

7. Vectors

- As directed line segments.
- Magnitude and direction of a vector.
- Types: equal vectors, unit vectors, zero vector.
- Position vector.
- Components of a vector.
- Vectors in two and three dimensions.
- $\hat{i}, \hat{j}, \hat{k}$ as unit vectors along the x, y and the z axes; expressing a vector in terms of the unit vectors.

- Operations: Sum and Difference of vectors; scalar multiplication of a vector.
- Section formula.
- Simple questions based on the above e.g. A line joining the mid point of any two sides of a triangle is parallel to the third side and half of it, concurrency of medians.

8. Co-ordinate geometry in 3-Dimensions

- As an extension of 2-D.
- Distance formula.
- Section and midpoint formula.
- Equation of x-axis, y-axis, z axis and lines parallel to them.
- Equation of xy - plane, yz - plane, zx - plane.
- Direction cosines, direction ratios.
- Angle between two lines in terms of direction cosines /direction ratios.
- Condition for lines to be perpendicular/parallel.

NOTE: Understanding of dot product of vectors is required.

SECTION C

9. Statistics

- Median - direct and by using the formula.
- Quartiles- direct and by using the formula.
- Deciles- direct and by using the formula.
- Percentiles - direct and by using the formula.
- Mode - graphically, direct method and by using the formula.
- Estimation of median/quartiles from Ogives.

NOTE: The following are also required to be covered:

- The Median, Quartiles, Deciles and Percentiles of grouped and ungrouped data;
- Mode – grouped and ungrouped data; estimation of mode by using graphical method. (Bimodal data not included).

10. Average Due Date

- Zero date.
- Equated periods.